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Heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation

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Abstract

A boundary layer problem on heat transfer in a viscoelastic boundary layer fluid flow over a non-isothermal porous sheet, where the flow is generated due to linear stretching of the sheet and influenced by a continuous suction/blowing of the fluid through the porous boundary, has been presented. In the flow region, heat balance is maintained with a temperature dependent heat source/sink, viscous dissipation and thermal radiation. Applying suitable similarity transformations on the highly non-linear momentum boundary layer equation and thermal boundary layer equation several closed from analytical solutions have been derived for non-dimensional temperature and heat flux profiles in the from of confluent hyper geometric (Kummer's) functions and other elementary functions as its special form. Heat transfer analysis has been carried out for two general types of boundary heating processes, namely, (i) prescribed quadratic power law surface temperature (PST) and (ii) prescribed quadratic power law surface heat flux (PHF) for various values of non-dimensional viscoelastic parameter k_1^*, Prandtl number Pr, Eckert number E, radiation parameter N, suction/blowing parameter v_w and source/sink parameter β. Some of the several important findings reported in this paper are (i) the combined effect of Prandtl number Pr, radiation parameter N and suction/blowing parameter v_w has significant impact in controlling the rate of heat transfer to the boundary layer region through the porous stretching sheet and (ii) radiation and suction can be used as means of cooling the viscoelastic boundary layer flow region. Special cases of our results are in excellent agreement with some of the existing work.

Keywords: Heat transfer; Viscoelastic fluid; Stretching sheet; Suction/blowing; Non-isothermal boundary; Radiation and heat generation

1. Introduction

Momentum and heat transfer in a viscoelastic boundary layer over a linear stretching sheet have been studied extensively in the recent past because of its ever-increasing usage in polymer processing industry, in particular in manufacturing process of artificial film and artificial fibers. In some applications of dilute polymer solution, such as the 5.4% solution of polyisobutylene in cetane, the viscoelastic fluid flow occurs over a stretching sheet [1,2]. Rajagopal et al. [2] have studied viscoelastic second order fluid flow over a stretching sheet by solving the momentum boundary layer equation numerically. Troy et al. [3] discussed uniqueness of the momentum boundary layer equation. Subsequently Chang [4] and Rao [5] showed the non-uniqueness of the solution and derived different forms of non-unique solution. All these works do not take into account the heat transfer phenomenon. Siddappa and Abel [6] have presented similar flow analysis without heat transfer in the flow of non-Newtonian fluid of the type Walters' liquid B. Although Lawrence and Rao [7] presented a work on heat transfer in the flow of viscoelastic fluid over a stretching sheet it did not consider viscous dissipation. However, viscoelastic fluid flow being a non-Newtonian fluid flow generates heat by means of viscous dissipation. There is another important aspect, which should also be taken into the account in a situation when there would be a temperature dependent heat source/sink present in the boundary layer region. In order to deal with both the situations Bujurke

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et al. [8] have presented a work on momentum and heat transfer in the second order viscoelastic fluid over a stretching sheet with internal heat generation and viscous dissipation. An exact analytical solution of MHD flow of a viscoelastic Walters liquid B past a stretching sheet has been presented by Andersson [9]. The effects of internal heat generation on heat transfer phenomenon are excluded from their analysis.

A new dimension is added to the study of viscoelastic boundary layer fluid flow and heat transfer by considering the effect of thermal radiation. Thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. The quality of the final product depends to a certain extent on the heat controlling factors. In view of this Raptis and Perdikis [10] analysed viscoelastic flow and heat transfer past a semi-infinite porous plate having constant suction of the fluid in presence of thermal radiation. Viscous dissipation which must be taken into account in the heat transfer analysis of non-Newtonian fluid flow is excluded from this study. Raptis [11] studied boundary layer flow and heat transfer of micropolar fluid past a continuously moving plate with viscous dissipation in the presence of radiation. Raptis [12] has also investigated the viscoelastic fluid flow past a semi-infinite plate taking into consideration of radiation using Rosseland approximation [13] when the free stream velocity and the temperature of the plate are not constant. However, this work does not deal with the situation when there would be a temperature dependent heat source/sink, viscous dissipation and suction/blowing through the porous boundary surface. Kumari and Nath [14] studied radiation effect in a non-Darcy mixed convection flow over a solid surface immersed in a saturated porous medium using Rosseland approximation. However, their study is confined to viscous fluid flow only. Siddheshwar and Mahabaleswar [15] studied MHD flow and heat transfer in a viscoelastic liquid over a stretching sheet with viscous dissipation, internal heat generation/absorption and radiation. This work does not take into account permeable stretching boundary condition.

Hence, in the present study we investigate the effect of thermal radiation on heat transfer in a boundary layer viscoelastic fluid flow over a semi-infinite porous stretching sheet taking into consideration of the viscous dissipation and temperature dependent heat source/sink. Radiation has been accounted in this study using Rosseland approximation [13].

We know that thermal boundary layer equation with viscous dissipation term is a non-homogeneous partial differential equation involving quadratic power of the velocity gradient. To seek a similarity solution of the thermal boundary layer equation, in case of linear stretching problem, we contemplate to deal with quadratic power law thermal boundary conditions for two general cases of boundary heating of the type (i) prescribed power law surface temperature of second degree (PST) and (ii) prescribed power law surface heat flux (PHF) of second degree. Several closed form analytical solutions for the heat transfer characteristics are obtained in the form of confluent hypergeometric function (Kummer's function). Solutions are also obtained in the form of some other elementary functions as the special cases of Kummer's function.

2. Governing basic equations and source and boundary conditions

2.1. Momentum boundary layer equation

Following the postulates of gradually fading memory, Coleman and Noll [16] derived the constitutive equation of second-order fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{2.1}$$

where T is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the dynamics viscosity, α_1, α_2 are the material moduli. A_1 and A_2 are the first two Rivlin–Ericksen tensors and they are defined as

$$A_1 = (\operatorname{grad} q) + (\operatorname{grad} q)^{\mathrm{T}} \tag{2.2}$$

$$A_2 = \frac{\mathrm{d}A_1}{\mathrm{d}t} + A_1(\operatorname{grad} q) + (\operatorname{grad} q)^{\mathrm{T}} \cdot A_1 \tag{2.3}$$

The model equation (2.1) was derived by considering up to second-order approximation of retardation parameter. Dunn and Fosdick [17] have given the range of values of material moduli μ, α_1 and α_2 as:

$$\mu \geqslant 0, \quad \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0 \tag{2.4}$$

The fluid modeled by Eq. (2.1) with the relationship (2.4) is compatible with the thermodynamics. The third relation is the consequence of satisfying the Clausius–Duhem inequality by fluid motion and the second relation arises due to the assumption that specific Helmholtz free energy of the fluid takes its minimum values in equilibrium. Later on Fosdick and Rajagopal [18] have reported , by using the data reduction from experiments, that in the case of a second-order fluid the material moduli μ, α_1 and α_2 should satisfy the relation.

$$\mu \geqslant 0, \quad \alpha_1 \leqslant 0, \quad \alpha_1 + \alpha_2 \neq 0 \tag{2.5}$$

They also reported that that the fluids modeled by Eq. (2.1) with the relationship (2.5) exhibit some anomalous behaviour. We must mention that second-order fluid, obeying model equation (2.1) with $\alpha_1 < \alpha_2$, $\alpha_1 < 0$ although exhibits some undesirable instability characteristics the second-order approximation is valid at low shear rate [2]. Now in literature the fluid satisfying the model equation (2.1) with $\alpha < 0$ is termed as second-order fluid and with $\alpha > 0$ is termed as second grade fluid [2].

We consider a laminar steady state incompressible viscoelastic second order fluid flow over a porous semi-infinite stretching sheet. The flow is generated as the consequence of linear stretching of the boundary sheet, caused by simultaneous application of equal and opposite forces along x-

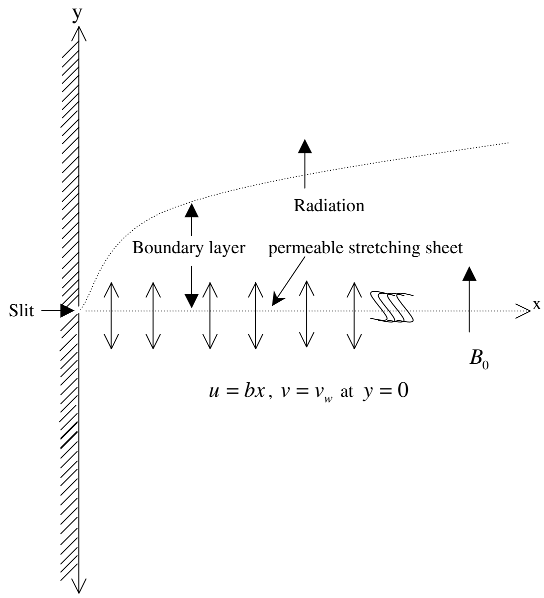


Fig. 1. Boundary layer over a permeable linear stretching sheet.

axis whilst keeping the origin fixed (Fig. 1). The governing boundary layer equations for momentum in such flow situations [19,2], in the usual form, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\,\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} \tag{2.7}$$

Here u and v are the velocity components in x and y directions, respectively, γ is the kinematic coefficient of viscosity, $k_0=\frac{-\alpha_1}{\rho}$ is the elastic parameter. Hence, in the case of second-order fluid flow k_0 takes positive value as α_1 takes negative value and other quantities have their usual meanings. In deriving Eq. (2.7) it is assumed that the normal stress is of the same order of magnitude as that of the shear stress, in addition to usual boundary layer approximations.

2.1.1. Boundary conditions on velocity

For the present physical problem, where the stretching of the boundary surface is assumed to be such that the flow directional velocity is linear function of the flow directional coordinate, we employ the following boundary condition [20]:

$$u=bx,\quad v=v_w,\quad \text{at } y=0 \tag{2.8}$$

$$u=0,\quad \text{as } y\to\infty \tag{2.9}$$

Here, the subscript y represents differentiation w.r.t y and b is the linear stretching rate constant. The constant v_w represents suction velocity across the stretching sheet when $v_w<0$, it is blowing velocity when $v_w>0$ and it represents impermeability of the wall when $v_w=0$.

2.2. Thermal boundary layer equation

We consider that the whole flow field is exposed under thermal radiation. In order to get the effect of temperature difference between the surface and the ambient fluid, we consider temperature dependent heat source/sink in the flow region. Since the fluid considered for analysis is viscoelastic the energy will be stored in the fluid by means of frictional heating due to viscous dissipation. So, we take into account of this. However, we assume that the fluid possesses strong viscous property in comparison with the elastic property. Also, the effect of elastic deformation terms might not be significant as the momentum boundary layer equation is valid at low shear rate and small values of elastic parameter [2]. Numerous works are also available in the literature of viscoelastic boundary layer flow which recognize this fact while studying heat transfer [10,12,15, 19–22]. In view of this discussion we may neglect the contribution of heat energy due to elastic deformation. Hence, the governing boundary layer equation for heat transfer takes the modified version of equations presented by Cortell [19] and Vajravelu and Soewono [21], as follows:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho c_p}(T-T_\infty) \tag{2.10}$$

where k is the thermal conductivity of the fluid, μ is the coefficient of viscosity of the fluid, T_∞ is the fluid temperature far away from the sheet and q_r is the radiative heat flux. The term Q represents the heat source when $Q>0$ and the heat sink when $Q<0$. Other quantities have their usual meanings [22].

Using Rosseland approximation for radiation [13] we can write

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.11}$$

Here, σ is the Stefan–Boltzmann constant and k^* is the absorption coefficient. Further we assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4\equiv 4T_\infty^3 T-3T_\infty^4 \tag{2.12}$$

Therefore, Eq. (2.10) is simplified to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho c_p}(T-T_\infty)+\frac{1}{3\rho c_p}\frac{16\sigma T_\infty^3}{k^*}\frac{\partial^2 T}{\partial y^2} \tag{2.13}$$

2.2.1. Boundary conditions on temperature

We intend to analyse the heat transfer phenomenon for two general types of boundary heating process, namely (i) prescribed power law surface temperature (PST) and (ii) prescribed power law heat flux (PHF).

In order to deal with non-isothermal stretching boundary *in PST case* we consider the appropriate boundary conditions on temperature as

$$T = T_w = T_\infty + A_0\left(\frac{x}{l}\right)^2 \quad \text{at } y = 0$$
$$T \to T_\infty \quad \text{as } y \to \infty \tag{2.14}$$

where T_w and T_∞ are temperature at wall and temperature far away from the wall, respectively. A_0 is a constant whose value depends on the properties of the fluid. The constant l is chosen as characteristic length. In order to obtain the closed form analytical solutions of the differential equation (2.13) we consider stretched boundary surface with prescribed power law temperature of second degree only.

In PHF case, the corresponding boundary conditions on temperature are

$$-k\left(\frac{\partial T}{\partial y}\right)_w = q_w = E_0\left(\frac{x}{l}\right)^2 \quad \text{at } y = 0$$
$$T \to T_\infty \quad \text{as } y \to \infty \tag{2.15}$$

where E_0 is a constant whose value depends on the properties of the fluid.

In this regard let us have a look on the typical choice of boundary conditions of the form (2.14) and (2.15). The thermal boundary layer equation (2.13) is a non-homogeneous equation of dependent variable T involving quadratic power of the velocity gradient. Therefore, in case of linear stretching problem, with a view to transform Eq. (2.13) into a similarity equation, we have chosen the temperature boundary conditions involving quadratic functions of x in the above forms (Eqs. (2.14) and (2.15)).

3. Solution of the momentum equation

Rajagopal et al. [2] have studied the flow of a viscoelastic fluid of the type Walters liquid B over an impervious stretching sheet. They used the following transformations

$$u = bxf_\eta(\eta), \quad v = -(b\gamma)^{1/2}f(\eta), \quad \eta = \sqrt{b/\gamma}\,y, \tag{3.1}$$

where f is the dimensionless stream function and η is the similarity variable. Substitution of Eq. (3.1) in Eq. (2.7) results in a forth order non-linear ordinary differential equation

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1^*\left\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\right\} \tag{3.2}$$

where $k_1^* = \frac{k_0 b}{\gamma}$ is the dimensionless viscoelastic parameter. The corresponding boundary conditions on f are of the form

$$f_\eta = 1, \quad f = -\frac{v_w}{\sqrt{b\gamma}} \quad \text{at } \eta = 0$$
$$f_\eta = 0 \quad \text{as } \eta \to \infty \tag{3.3}$$

It is to be noted that the boundary conditions prescribed by Eq. (3.3) are not sufficient to solve the problem (3.2) uniquely. A critical review on the boundary conditions and the existence and uniqueness of the solution have been given by Rajagopal [23]. Most of the available literature on boundary layer flow of a viscoelastic fluid over linearly stretching sheets deal with the three boundary conditions on velocity, which are one less than the number required to solve the problem uniquely [2,24,9,19,25]. The augmentation of the boundary condition has also been discussed in the work of Rajagopal and Gupta [26].

Making use of the boundary conditions (3.3) with $v_w = 0$, Rajagopal et al. [2] obtained the corresponding solution of Eq. (3.2). Subsequently Mcleod and Rajagopal [27] and Troy et al. [3] obtained unique solution of Eq. (3.2) in the form

$$f(\eta) = 1 - e^{-\eta} \quad \text{when } k_1^* = 0 \tag{3.4}$$

Also, for $0 < k_1^* < 1$, Troy et al. [3] found a solution in the form

$$f(\eta) = \sqrt{1 - k_1^*}\left(1 - e^{-\frac{\eta}{\sqrt{1-k_1^*}}}\right) \tag{3.5}$$

Later on Chang [4] showed that the solution of Eq. (3.2) satisfying the boundary conditions of Eq. (3.3) along with $v_w = 0$ is not unique. Taking $k_1^* = 1/2$ Chang [4] presented another solution of the form

$$f(\eta) = \sqrt{2}\left[1 - e^{-\frac{\eta}{\sqrt{2}}}\cos\left(\frac{\sqrt{3}}{2}\eta\right)\right] \tag{3.6}$$

Recently Rao [5] derived another closed from solution of the from

$$f(\eta) = A\left[1 - e^{-A\eta/2}\left\{\cos\left(\sqrt{3}A\eta\right) + \frac{(1+2k_1)}{\sqrt{3}}\sin(\sqrt{3}A\eta/2)\right\}\right]$$
$$A = \frac{1}{\sqrt{-k_1}}, \quad k_1 = -k_1^*, \tag{3.7}$$

The above form of solution exists only for $k_1 \in (-1, 0)$. Lawrence and Rao [28] presented a general method and obtained all the non-unique solutions of the modified equation of (3.2) with transverse magnetic field.

Among all these solutions of the form (3.5) is the realistic one as we can recover the Navier–Stokes solution only in its limiting case $k_1^* = 0$ and for slightly viscoelastic fluid , assigning small value of elastic parameter in the equation, we get a boundary layer only slightly altered in its dimensions from the viscous one [7]. Moreover for viscoelastic fluid where k_1^* should be real positive the solution (3.5) is the only realistic type of solution of the problem. Following this analysis we derived the realistic solution of Eq. (3.2) using the given boundary conditions in the form

$$f(\eta) = \frac{1 - \exp(-\alpha\eta)}{\alpha} - \frac{v_w}{\sqrt{bv}} \tag{3.8}$$

where α is a real positive root of the cubic algebraic equation

$$\alpha^3 + \frac{1-k_1^*}{v_w \frac{k_1^*}{\sqrt{b\gamma}}}\alpha^2 + \frac{1}{k_1^*}\alpha - \frac{1}{v_w \frac{k_1^*}{\sqrt{b\gamma}}} = 0 \tag{3.9}$$

The limiting case of the expression (3.9) when $v_w = 0$ yields the result of Andersson [9] in absence of magnetic field. The quadratic equation for α, in such a case, may be deduced from Eq. (3.9) in the limit $v_w \to 0$.

4. Solutions of heat transfer equation

4.1. Case A: prescribed power law surface temperature (PST)

In the PST case we define non-dimensional temperature variable as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{4.1}$$

where the expression for $T_w - T_\infty$ is given in Eq. (2.14). Now, we make use of the transformations given by Eqs. (3.1) and (4.1) in the Eq. (2.13). This leads to the non-dimensional from of temperature equation as follows:

$$\theta_{\eta\eta} + \frac{3PrN}{(3N+4)} f\theta_\eta - \frac{3PrN}{3N+4)}(2f_\eta - \beta)\theta = -\frac{3PrN}{(3N+4)} Ef_{\eta\eta}^2 \tag{4.2}$$

where $Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $\beta = \frac{Q}{\rho c_p b}$ is the heat source/sink parameter, $E = \frac{b^2 l^2}{A_0 c_p}$ is the Eckert number and $N = \frac{kk^*}{4\sigma T_\infty^3}$ is the radiation parameter.

Using the dimensionless variable of Eq. (4.1) in Eq. (2.14) we get the corresponding dimensionless boundary conditions as

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{4.3}$$

Defining new variables

$$\xi = -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right) e^{-\alpha\eta} \tag{4.4}$$

Before obtaining the solution of Eq. (4.2) we must provide the explicit form of solution of $f(\eta)$. Making use of the solution (3.8) and transformation given by Eq. (4.4) in Eq. (4.2), we derive the governing equation for temperature, in the form

$$\xi\theta_{\xi\xi} + (1 - a_0 - \xi)\theta_\xi + \left(2 + \frac{Pr\beta}{\alpha^2\xi}\frac{3N}{3N+4)}\right)\theta = -\frac{E\alpha^4}{Pr}\left(\frac{3N+4}{3N}\right)\xi \tag{4.5}$$

$$\text{where,} \quad a_0 = \left(\frac{Pr}{\alpha^2} - \frac{Pr}{\alpha}\frac{v_w}{\sqrt{b\gamma}}\right)\left(\frac{3N}{3N+4}\right) \tag{4.6}$$

The corresponding boundary conditions are

$$\theta\left(-\frac{Pr}{\alpha^2}\frac{3N}{3N+4}\right) = 1, \quad \theta(0) = 0 \tag{4.7}$$

The Eqs. (4.5) and (4.7) constitute a non-homogeneous boundary value problem. Denoting the solution of the homogeneous part of Eq. (4.5) by θ_c and further introducing the transformation

$$\theta_c = \xi^{\delta_1}\omega(\xi)$$

we obtain the confluent hypergeometry equation of the form

$$\xi\omega_{\xi\xi} + (1 + b_0 - \xi)\omega_\xi - \tfrac{1}{2}(a_0 + b_0 - 4)\omega = 0 \tag{4.8}$$

where

$$\delta_1 = \frac{a_0 \pm b_0}{2} \quad \text{and} \quad b_0 = \sqrt{a_0^2 - \frac{4Pr\beta}{\alpha^2}\left(\frac{3N}{3N+4}\right)} \tag{4.9}$$

The relation $\frac{4Pr\beta}{\alpha^2}\left(\frac{3N}{3N+4}\right) \leqslant a_0^2$ must be satisfied in order to have real values of b_0.

The solution of Eq. (4.8) is

$$\omega = M\left(\frac{a_0 + b_0 - 4}{2}, 1 + b_0, \xi\right) \tag{4.10}$$

where M is the Kummer's function (Abramowitz and Stegun [29] and it is defined by

$$M(a_0, b_0, z) = 1 + \sum_{n=1}^{\infty}\frac{(a_0)_n z^n}{(b_0)_n n!}$$

$$(a_0)_n = a_0(a_0+1)(a_0+2), \ldots, (a_0+n-1)$$
$$(b_0)_n = b_0(b_0+1)(b_0+2), \ldots, (b_0+n-1) \tag{4.11}$$

The particular integral of Eq. (4.5) is

$$\theta_p(\xi) = -\frac{\alpha^4 E}{Pr}\frac{\left(\frac{3N+4}{3N}\right)}{Pr\left(4 - 2a_0 + \frac{Pr\beta}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)\xi^2} \tag{4.12}$$

Hence, the solution of Eq. (4.5) is

$$\theta(\xi) = a_1\theta_c(\xi) + \theta_p(\xi) \tag{4.13}$$

Now making use of the boundary conditions of Eq. (4.7) and changing the variable ξ to η we obtain the solution in the following form of confluent hypergeometric function

$$\theta(\eta) = \left(1 - \frac{c_1 Pr^2}{\alpha^4}\right) e^{-\left(\frac{a_0+b_0}{2}\right)\alpha\eta} \times \frac{M\left(\frac{a_0+b_0}{2} - 2, 1 + b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)e^{-\alpha\eta}\right)}{M\left(\frac{a_0+b_0}{2} - 2, 1 + b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)} + c_1\left(\frac{Pr}{\alpha^2}\right)^2 e^{-2\alpha\eta} \tag{4.14}$$

where

$$c_1 = -\frac{E\alpha^4}{Pr}\left(\frac{3N}{3N+4}\right)\frac{1}{\left(4 - 2a_0 + \frac{Pr\beta}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)} \tag{4.15}$$

a_0 and b_0 are given by Eqs. (4.6) and (4.9), respectively.

Dimensionless wall temperature gradient $\theta_\eta(0)$ is obtained as

$$\theta_\eta(0) = A\left\{\frac{Pr}{2\alpha}\left(\frac{3N}{3N+4}\right)\frac{(a_0+b_0-4)}{(1+b_0)} \times \frac{M\left(\frac{(a_0+b_0-2)}{2}, 2+b_0, \frac{-Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)}{M\left(\frac{(a_0+b_0-4)}{2}, 1+b_0, \frac{-Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)} - \alpha\frac{(a_0+b_0)}{2}\right\} - 2\alpha c_1\frac{Pr^2}{\alpha^4} \quad (4.16)$$

where $A = 1 - c_1\frac{Pr^2}{\alpha^4}$.

We know that the Kummer's function is related to other special forms of elementary functions. One of such special forms [29] may be obtained by assigning special value to its argument. Hence, setting $\frac{a_0+b_0-4}{2} = 1+b_0$ we deduce the expression for non-dimensional temperature profile in the special form

$$\theta(\eta) = \left\{1+\frac{EPr}{(a_0-5)}\right\}\exp\left[(3-a_0)\alpha\eta + \frac{Pr}{\alpha^2}\frac{3N}{(3N+4)}(1-e^{-\alpha\eta})\right] - \frac{EPr}{(a_0-5)}\exp(-2\alpha\eta) \quad (4.17)$$

Expression for non-dimensional wall temperature gradient $\theta_\eta(0)$ in the special form is deduced as

$$\theta_\eta(0) = \left\{1+\frac{EPr}{(a_0-5)}\right\}\left[3\alpha - a_0\alpha + \frac{Pr}{\alpha}\frac{3N}{(3N+4)}\right] + \frac{2\alpha EPr}{(a_0-5)} \quad (4.18)$$

Dimensional local heat flux q_w is defined as

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = k\sqrt{\tfrac{b}{\gamma}}(T_w - T_\infty)[-\theta_\eta(0)] \quad (4.19)$$

$$c_2 = \frac{\left\{2c_1\alpha\left(\frac{Pr}{\alpha^2}\right)^2 - 1\right\}\left(\frac{3N+4}{3N}\right)}{\left[-\alpha\left(\frac{a_0+b_0}{2}\right)\left(\frac{3N+4}{3N}\right)M\left(\frac{a_0+b_0}{2}-2, 1+b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right) + \frac{(a_0+b_0-4)}{2(1+b_0)}\frac{Pr}{\alpha}M\left(\frac{a_0+b_0}{2}-1, 2+b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right)\right]} \quad (4.26)$$

4.2. Case B: prescribed power law heat flux (PHF)

In PHF case we define dimensionless new temperature variable as

$$g(\eta) = \frac{T - T_\infty}{E_0\left(\frac{x}{l}\right)^2\frac{1}{k}\sqrt{\frac{\gamma}{b}}} \quad (4.20)$$

and make use of the transformations given by Eqs. (3.1). This leads to the following non-dimensional form of Eq. (2.13) for temperature.

$$g_{\eta\eta} + Pr\left(\frac{3N}{3N+4}\right)fg_\eta - Pr\left(\frac{3N}{3N+4}\right)(2f_\eta - \beta)g = -Pr\left(\frac{3N}{3N+4}\right)Ef_{\eta\eta}^2 \quad (4.21)$$

Corresponding boundary conditions in PHF case are

$$g_\eta(0) = -1, \quad g(\infty) = 0 \quad (4.22)$$

In order to solve Eq. (4.21) we must provide the explicit form of solution of $f(\eta)$.

Making use of the solution given by Eqs. (3.8) and (3.9) and the transformation of Eq. (4.4) we deduce the transformed basic equation and boundary conditions of temperature, in the following form:

$$\xi g_{\xi\xi} + (1-a_0-\xi)g_\xi + \left(2+\frac{\beta Pr}{\alpha^2\xi}\left(\frac{3N}{3N+4}\right)\right)g = -\frac{E\alpha^4}{Pr}\left(\frac{3N+4}{3N}\right)\xi \quad (4.23)$$

$$g_\xi\left(-\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right) = -\frac{\alpha}{Pr}\left(\frac{3N+4}{3N}\right), \quad g(0) = 0 \quad (4.24)$$

The analytical solution of Eq. (4.23), subject to the corresponding boundary conditions of Eq. (4.24), is obtained in the following form of confluent hypergeometric function of the similarity variable η.

$$g(\eta) = c_2 e^{-\left(\frac{a_0+b_0}{2}\right)\alpha\eta}M\left(\frac{a_0+b_0}{2}-2, 1+b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)e^{-\alpha\eta}\right) + c_1 e^{-2\alpha\eta}\left(\frac{Pr}{\alpha^2}\right)^2 \quad (4.25)$$

where

and expression for c_1 is given by Eq. (4.15). The expressions for a_0 and b_0 are given by Eqs. (4.6) and (4.9), respectively.

The expression for dimensionless wall temperature is obtained as

$$g(0) = c_2 M\left(\frac{a_0+b_0}{2}-2, 1+b_0, -\frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)\right) + c_1\left(\frac{Pr}{\alpha^2}\right)^2 \quad (4.27)$$

The limiting cases of our results, obtained from Eqs. (4.25)–(4.27), produce the corresponding results of Sonth et al. [30] when $N \to \infty$.

The special form of the result may be deduced from Eq. (4.25) by setting $\frac{a_0+b_0-4}{2} = 1+b_0$ in the following form:

$$g(\eta) = \frac{\left\{1 + \frac{2\alpha E Pr}{(a_0 - 5)}\right\}}{\alpha(a_0 - 3) - \frac{Pr}{\alpha}\left(\frac{3N}{3N+4}\right)} \times \exp\left[(3 - a_0)\alpha\eta + \frac{Pr}{\alpha^2}\left(\frac{3N}{3N+4}\right)(1 - e^{-\alpha\eta})\right] - \frac{E Pr}{(a_0 - 5)} \exp[-2\alpha\eta] \tag{4.28}$$

Expression for non-dimensional wall temperature $g(0)$ in the special form is deduced as

$$g(0) = \frac{\left\{1 + \frac{2\alpha E Pr}{(a_0 - 5)}\right\}}{\alpha(a_0 - 3) - \frac{Pr}{\alpha}\left(\frac{3N}{3N+4}\right)} - \frac{E Pr}{(a_0 - 5)} \tag{4.29}$$

The expressions for wall temperature in dimensional form is

$$T_w = T_\infty + \frac{E_0}{k}\left(\frac{x}{l}\right)^2 \sqrt{\frac{\gamma}{b}} g(0) \tag{4.30}$$

5. Discussion of the results

A boundary layer problem for momentum and heat transfer in a viscoelastic fluid flow over a non-isothermal porous stretching sheet, in the presence of thermal radiation, is examined in this paper. Linear stretching of the porous boundary, viscous dissipation, temperature dependent heat source/sink and thermal radiation are taken into consideration in this study. The basic boundary layer partial differential equations, which are highly non-linear, have been converted into a set of non-linear ordinary differential

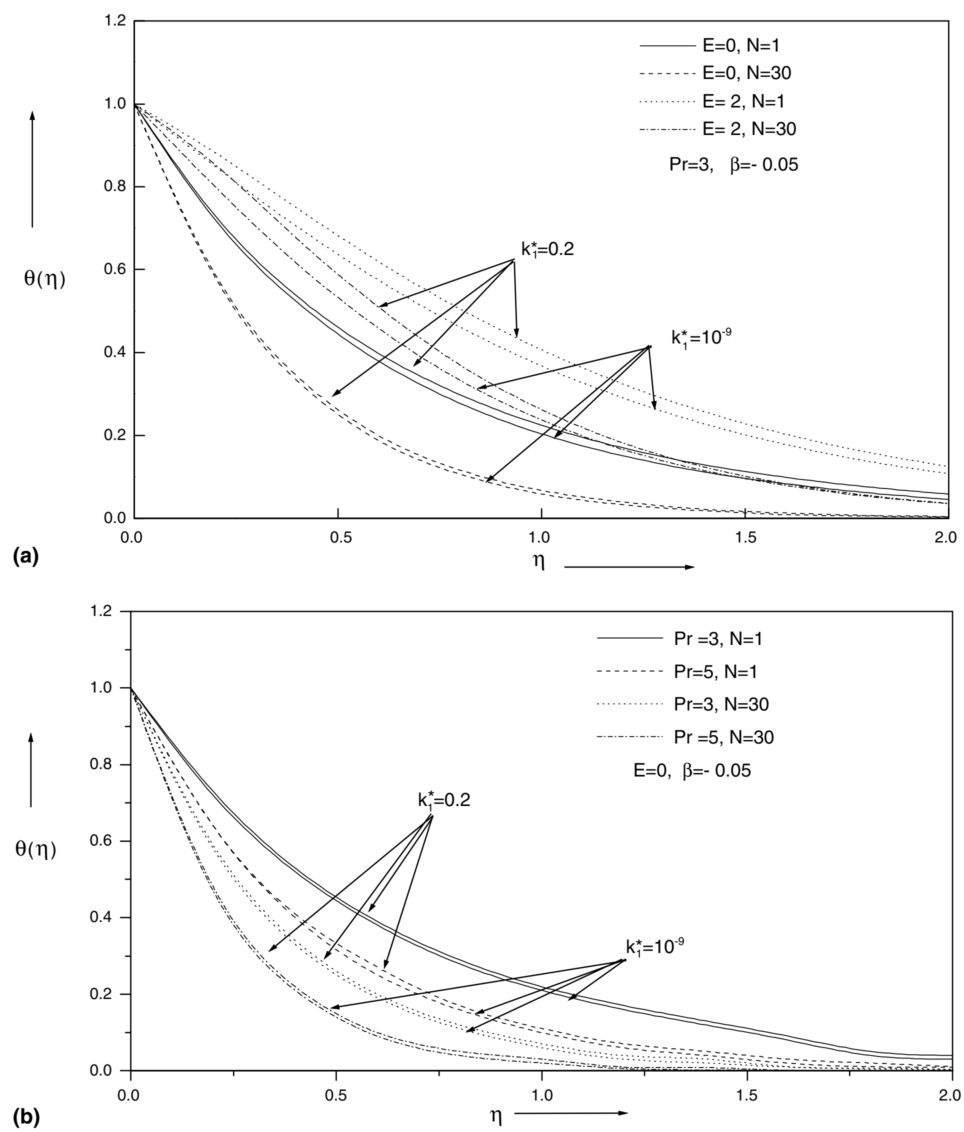


Fig. 2. Dimensionless temperature profiles $\theta(\eta)$ for various values of Eckert number E (a), Prandtl number Pr (b) and radiation parameter N, when $v_w = -0.04$ in PST case.

equations by applying suitable similarity transformations and their analytical solutions are obtained in terms of confluent hypergeometric function (Kummer's function). Different analytical expressions are obtained for non-dimensional temperature profile for two general cases of boundary conditions, namely: (i) prescribed second order power law surface temperature (PST) and (ii) prescribed second order power law heat flux (PHF). Explicit analytical expressions are also obtained for dimensionless temperature gradient $\theta_\eta(0)$ and local heat flux q_w for general cases as well as special cases of different physical situations. Since, the present problem is the extension of our earlier problem [30] to the case of a flow region emitting thermal radiation we intend to restrict our analysis for the effect of thermal radiation on various heat transfer characteristics in different physical situations of viscous dissipation, viscoelasticity, heat source/sink, suction/blowing through porous boundary and impermeability of the wall. Numerical computations of the results are demonstrated in the Figs. 2–4 for PST and PHF cases, respectively. Results for wall temperature gradient in PST case and wall temperature in PHF case are recorded in Tables 1a, 2a and 1b, 2b, respectively in order to have greater insight in the qualitative analysis of the results. In the process of computation we solve the cubic algebraic Eq. (3.9) for α using Graffe's root squaring method. Through out computation we assign $b = 2$ and $\gamma = 0.04$.

Fig. 2a is drawn for temperature profile for various values of Eckert number E and radiation parameter N. From this graph we notice that the effect of radiation parameter N is to decrease temperature throughout the boundary layer flow field. The increase of radiation parameter N implies the release of heat energy from the flow region by mean of radiation, whereas, the effect of Eckert number E and viscoelastic parameter k_1^* is to increase the temperature. The combined effect of increasing values of viscoelastic parameter k_1^* and Eckert number E is to enhance temperature significantly in the flow field. This is quite consistent with the fact that viscoelastic fluid, being a non-Newtonian fluid, experiences tensile stress and frictional heating in the layer. These contribute thickening of thermal boundary layer and results in increase of temperature in the

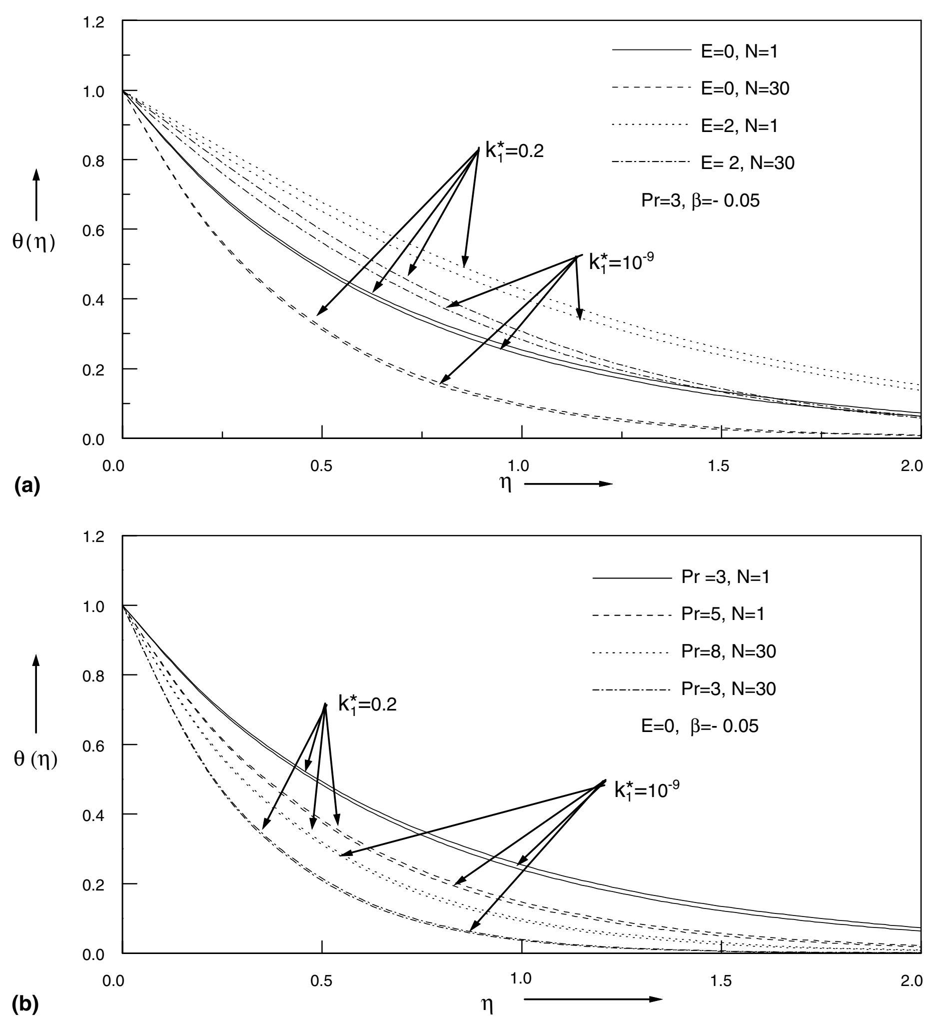
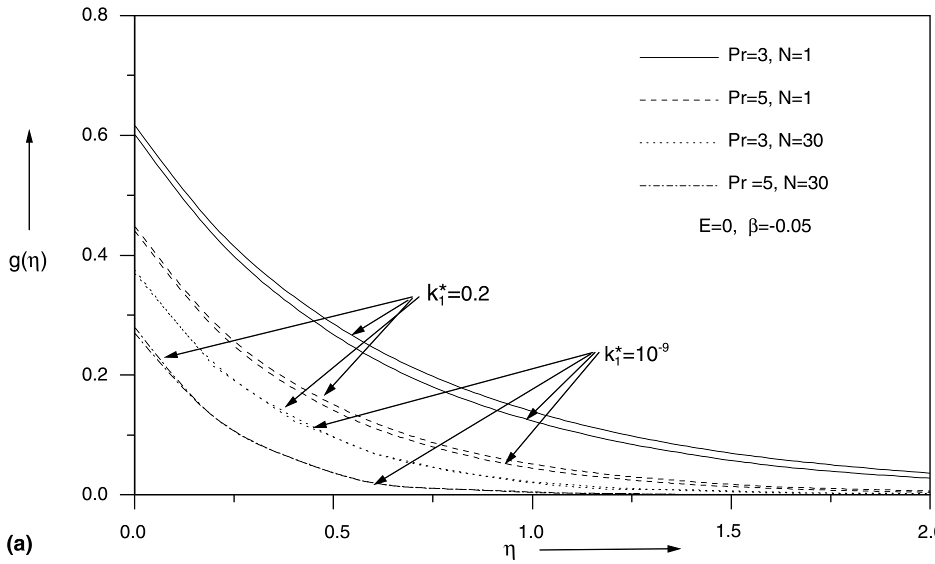


Fig. 3. Dimensionless temperature profiles $\theta(\eta)$ for various values of Eckert number E (a), Prandtl number Pr (b) and radiation parameter N, when $v_w = 0.04$ in PST case.



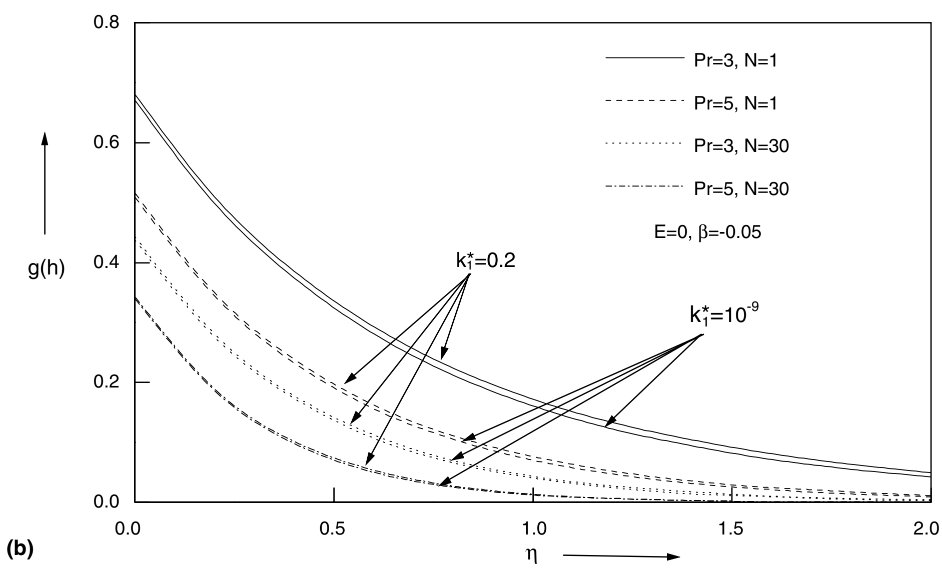


Fig. 4. Dimensionless temperature profile $g(\eta)$ for various values of Prandtl number Pr and radiation parameter N, when $v_w = -0.04$ (a) and $v_w = 0.04$ (b) in PHF case.

boundary significantly. Fig. 2b depicts the graphical representation of temperature profile for various values of Prandtl number Pr and radiation parameter N. The effect of increasing values of Prandtl number Pr is to decrease temperature at a point in the flow field, as there would be a thinning of the thermal boundary layer as a result of reduced thermal conductivity. The combined effect of increasing values of Pr and N is to reduce temperature largely in the boundary layer flow region. The comparative study of the Fig. 2a and b reveals that the effect of presence of frictional heating ($E \neq 0$) is to enhance temperature in the fluid significantly near the stretching sheet, which is in conformity with the realistic situation.

Fig. 3a and b are plotted for the same data sets as those of Fig. 2a and b, respectively except for $v_w = 0.04$. In these cases temperature profiles follow similar patterns as in the case of Fig. 2a and b, respectively. However, the comparative study of Figs. 3 and 4 reveals that the effect of blowing parameter $v_w = 0.04$ is to increase the temperature throughout the boundary layer except on the wall where it attains unity in PST case. This is due to the fact that thermal boundary layer increases in the case of blowing. It is noticeable that the reduction of temperature due to the radiation is more prominent in the case of suction than that of the blowing. Fig. 4a demonstrates the dimensionless temperature profile $g(\eta)$ for various values Prandtl number Pr and radiation parameter N when their will be a suction of fluid through porous boundary in the PHF case. Fig. 4b is drawn for same set of data as that of Fig. 4a except for the case when there will be a blowing of fluid in the boundary layer region through the porous boundary. Qualitative behaviours of temperature profile in these cases follow the similar pattern as in the case of PST. Here, wall temperature will be different for different physical situations in

Table 1a
Wall temperature gradient $\theta_\eta(0)$ for the case of prescribed surface temperature (PST) when $\beta = -0.05$

k_1^*	Pr	N	E	$\theta_\eta(0)$ ($v_w = -0.04$)	$\theta_\eta(0)$ ($v_w = 0.0$)	$\theta_\eta(0)$ ($v_w = 0.04$)
10^{-9}	3	1	0	−1.66	−1.57	−1.49
0.2				−1.61	−1.54	−1.47
10^{-9}	5			−2.26	−2.11	−1.96
0.2				−2.22	−2.08	−1.94
10^{-9}	3	30		−2.70	−2.48	−2.28
0.2				−2.66	−2.45	−2.26
10^{-9}	5			−3.65	−3.28	−2.94
0.2				−3.61	−3.25	−2.92
10^{-9}	3	1	2	−0.75	−0.77	−0.78
0.2				−0.51	−0.59	−0.66
10^{-9}	5			−0.91	−0.92	−0.94
0.2				−0.55	−0.68	−0.77
10^{-9}	3	30		−0.99	−1.01	−1.02
0.2				−0.55	−0.71	−0.82
10^{-9}	5			−1.16	−1.17	−1.18
0.2				−0.49	−0.73	−0.89

Table 1b
Wall temperature $g(0)$ for the case of prescribed heat flux (PHF) when $\beta = -0.05$

k_1^*	Pr	N	E	$g(0)$ ($v_w = -0.04$)	$g(0)$ ($v_w = 0.0$)	$g(0)$ ($v_w = 0.04'$)
10^{-9}	3	1	0	0.60	0.63	0.67
0.2				0.62	0.65	0.68
10^{-9}	5			0.44	0.47	0.51
0.2				0.45	0.48	0.51
10^{-9}	3	30		0.37	0.40	0.44
0.2				0.38	0.41	0.44
10^{-9}	5			0.27	0.30	0.34
0.2				0.28	0.31	0.34
10^{-9}	3	1	2	1.15	1.14	1.14
0.2				1.30	1.26	1.23
10^{-9}	5			1.04	1.03	1.03
0.2				1.20	1.15	1.12
10^{-9}	3	30		1.00	0.99	0.99
0.2				1.17	1.12	1.08
10^{-9}	5			0.96	0.95	0.94
0.2				1.14	1.08	3.04

Table 2a
Wall temperature gradient $\theta_\eta(0)$ for the case of prescribed surface temperature (PST) when $\beta = 0.05$

k_1^*	Pr	N	E	$\theta_\eta(0)$ ($v_w = -0.04$)	$\theta_\eta(0)$ ($v_w = 0.0$)	$\theta_\eta(0)$ ($v_w = 0.04$)
10^{-9}	3	1	0	−1.61	−1.52	−1.44
0.2				−1.56	−1.49	−1.41
10^{-9}	5			−2.20	−2.05	−1.90
0.2				−2.16	−2.02	−1.87
10^{-9}	3	30		−2.63	−2.41	−2.21
0.2				−2.59	−2.38	−2.19
10^{-9}	5			−3.57	−3.19	−2.85
0.2				−3.53	−3.16	−2.83
10^{-9}	3	1	2	−0.68	−0.70	−0.72
0.2				−0.43	−0.52	−0.59
10^{-9}	5			−0.82	−0.84	−0.85
0.2				−0.45	−0.59	−0.68
10^{-9}	3	30		−0.89	−0.92	−0.93
0.2				−0.44	−0.61	−0.72
10^{-9}	5			−1.02	−1.04	−1.01
0.2				−0.34	−0.59	−0.76

Table 2b
Wall temperature $g(0)$ for the case of prescribed heat flux (PHF) when $\beta = 0.05$

k_1^*	Pr	N	E	$g(0)$ ($v_w = -0.04$)	$g(0)$ ($v_w = 0.0$)	$g(0)$ ($v_w = 0.04$)
10^{-9}	3	1	0	0.62	0.66	0.69
0.2				0.63	0.67	0.70
10^{-9}	5			0.45	0.48	0.52
0.2				0.46	0.49	0.53
10^{-9}	3	30		0.37	0.41	0.45
0.2				0.38	0.42	0.46
10^{-9}	5			0.27	0.31	0.34
0.2				0.28	0.32	0.35
10^{-9}	3	1	2	1.19	1.19	1.19
0.9				1.36	1.31	1.28
10^{-9}	5			1.07	1.07	1.07
0.2				1.25	1.20	1.16
10^{-9}	3	30		1.03	1.03	1.02
0.2				1.21	1.16	1.12
10^{-9}	5			0.99	0.98	0.97
0.2				1.18	1.12	1.08

contrast to the cases of PST where it attains the value unity. This behaviour of temperature profile is the consequence of prescribed boundary conditions.

The values of wall temperature gradient $\theta_\eta(0)$ when there would be a temperature dependent heat sink present in the flow field in the PST case are recorded in the Table 1a. From the table we notice that the effect of suction parameter v_w is to increase the numerical value of wall temperature gradient and that of blowing parameter is to decrease the same for the same set of values of viscous elastic parameter k_1^*, Prandtl number Pr and radiation parameter N in the absence of frictional heating ($E = 0$). However in the presence of frictional heating ($E = 2$) suction/blowing parameters have the reverse effect on the wall temperature gradient. Wall temperature in PHF case, for the same set of data as those of Table 1a, are recorded in the Table 1b. From this table we notice that the effect increasing values of Prandtl number Pr and radiation parameter N is to lower the wall temperature. Whereas the effect of increasing values of viscoelastic parameter k_1^* and Eckert number E is to increase wall temperature due to heat addition by means of frictional heating. The effect of suction parameter ($v_w < 0$) is to reduce wall temperature and that of blowing is to increase the same in the absence of frictional heating ($E = 0$). Whereas, in the presence of frictional heating suction/blowing parameters v_w have the reverse impact on the wall temperature in PHF case.

In order to know the impact of temperature dependent heat source/sink on heat transfer rate in PST case we have computed wall temperature gradient for the same data values of k_1^*, Pr, N, E as those in Table 1a expect for $\beta = 0.04$ and recorded in the Table 2a. Comparison study of the tabulated results of Tables 1a and 2a demonstrates that the effect of heat source parameter β is to lower the numerical value of wall temperature gradient $\theta_\eta(0)$, resulting in a reduced magnitude of heat transfer rate. This is quite consistent with the fact that the thermal boundary layer develops in the presence of heat source. Table 2a and 2b record wall temperature gradient and wall temperature, respectively for the same data set as those of Table 1a and 1b, respectively except for $\beta = 0.04$. Comparative study of Tables 1b and 2b provides information that wall temperature will rise in the presence of heat source in boundary layer region when the wall is maintained with prescribed heat flux.

6. Conclusion

A mathematical model study on the influence of thermal radiation in a viscoelastic boundary layer flow field over an acceleration stretching sheet, where flow is subject to suction/blowing through the porous boundary, has been carried out. Linear stretching of the porous boundary, viscous dissipation, temperature dependent heat source/sink, thermal radiation are taken into consideration in this study. Analytical solutions of the governing boundary layer partial differential equations, which are highly nonlinear, have been obtained in terms of confluent hypergeometry function (Kummer's function) and its special forms. Different analytical expressions are obtained for non-dimensional temperature profile for two general cases of boundary conditions, namely (i) prescribed second order power law surface temperature (PST) and (ii) prescribed second order power law heat flux (PHF). Explicit analytical expressions are also obtained for dimensionless temperature gradient $\theta_\eta(0)$ and local heat flux q_w for general cases as well as for special cases of different physical situations.

The specific conclusions derived from this study can be listed as follows:

(i) Explicit expressions are obtained for various heat transfer characteristics in the form of confluent hypergeometric functions (Kummer's function). Several expressions are also obtained in the form of some other elementary functions as the special cases of Kummer's function.

(ii) The combined effect of increasing values of viscoelastic parameter k_1^* and Eckert number E in the absence of radiation is to increase temperature significantly in the boundary layer flow field.

(iii) The combined effect of increasing the values of Prandtl number Pr and radiation parameter N is to reduce the temperature largely in the boundary layer flow region.

(iv) The effect of radiation parameter N is to reduce temperature significantly in the flow region when there would be suction of fluid through the porous stretching boundary. Hence the radiation and suction can be used as means of cooling the boundary layer region.

(v) The effect of the suction parameter v_w is to increase the numerical value of wall temperature gradient and that of blowing is to decrease the same for all values of small viscoelastic parameter k_1^*, Prandtl number Pr and radiation parameter N in absence of frictional heating ($E = 0$).

(vi) The limiting cases of the results of this paper are in excellent agreement with the results of Sonth et al. [30].

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References

[1] H. Markovitz, B.D. Coleman, Adv. Appl. Mech. 8 (1964).
[2] K.R. Rajagopal, T.Y. Na, A.S. Gupta, Flow of viscoelastic fluid over a stretching sheet, Rheol Acta 23 (1984) 213–215.
[3] W.C. Troy, E.A. Overman, H.G.B. Ermentrout, J.P. Keener, Uniqueness of flow of a second order fluid past a stretching sheet, Quart. Appl. Math. 44 (1987) 753–755.
[4] W.D. Chang, The non-uniqueness of the flow of a viscoelastic fluid over a stretching sheet, Quart. Appl. Math. 47 (20) (1989) 365–366.
[5] B.N. Rao, Technical Note: Flow of a fluid of second grade over a stretching sheet, Int. J. Non-Linear Mech. 31 (4) (1996) 547–550.
[6] B. Siddappa, M.S. Abel, Non-Newtonian flow past a stretching plate, Z Angew Math. Phys. 36 (1985) 890–892.
[7] P.S. Lawrence, B.N. Rao, Heat transfer in the flow of a viscoelastic fluid over a stretching sheet, Acta Mech. 93 (1992) 53–61.
[8] N.M. Bujurke, S.N. Biradar, P.S. Hiremath, Second order fluid flow past a stretching sheet with heat transfer, Z Angew Math. Phys. 38 (1987) 653–657.
[9] H.I. Andersson, Note: MHD flow of a viscoelastic fluid past a stretching surface, Acta Mech. 95 (1992) 227–230.
[10] A. Rapits, C. Perdikis, Viscoelastic flow by the presence of radiation, ZAAM 78 (4) (1998) 277–279.
[11] A. Rapits, Technical note; Flow of a micropolar fluid past a continuously moving plate by the presence of radiation, Int. J. Heat Mass Transfer 41 (1998) 2865–2866.
[12] A. Raptis, Radiation and viscoelastic flow, Int. Comm. Heat Mass Transfer 26 (6) (1999) 889–895.
[13] M.Q. Brewster, Thermal Radiative Transfer Properties, John Wiley and Sons, 1972.
[14] M. Kumari, G. Nath, Radiation effect on mixed convection from a horizontal surface in a porous medium, Mech. Res. Comm. 31 (2004) 483–491.
[15] P.G. Siddheshwar, U.S. Mahabaleswar, Effect of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet, Int. J. Non-Linear Mech. 40 (2005) 807–820.
[16] B.D. Coleman, W. Noll, An approximation theorem for functionals with applications in continuous mechanics, Arch. Ration. Mech. Anal. 6 (1960) 355–370.
[17] J.E. Dun, R.L. Fosdick, Thermodynamics stability and boundedness of fluids of complexity 2 and fluids of second grade, Arch. Ration. Mech. Anal. 56 (1974) 191–252.
[18] R.L. Fosdick, K.R. Rajagopal, Anomalous features in the model of second order fluids, Arch. Ration. Mech. Anal. 70 (1979) 145–152.
[19] R. Cortell, Similarity solutions for flow and heat transfer of a viscoelastic fluid over a stretching sheet, Int. J. Non-Linear Mech. 29 (2) (1994) 155–161.
[20] B.S. Dandapat, S.N. Sing, R.P. Sing, Heat transfer due to permeable stretching wall in presence of transverse magnetic field, Arch. Mech. 56 (2) (2004) 141–172.

[21] K. Vajravelu, E. Soewono, Fourth order nonlinear systems arising in combined free and forced convection flow of a second order fluid, Int. J. Non-Linear Mech. 29 (6) (1994) 861–869.
[22] B.S. Dandapat, A.S. Gupta, Flow and heat transfer in a viscoelastic fluid over a stretching sheet, Int. J. Non-Linear Mech. 24 (3) (1989) 215–219.
[23] K.R. Rajagopal, On the boundary conditions for fluids of the differential type, in: A. Sequeira (Ed.), Navier–Stokes Equation and Related Non-linear Problems, Plenum Press, New York, 1995, pp. 273–278.
[24] D. Rollins, K. Vajravelu, Heat transfer in a second-order fluid over a continuous stretching surface, Acta. Mech. 89 (1991) 167–178.
[25] T.R. Mahapatra, A.S. Gupta, Stagnation-point flow of a viscoelastic fluid towards a stretching surface, Int. J. Non-Linear Mech. 39 (2004) 811–820.
[26] K.R. Rajagopal, A.S. Gupta, An exact solution for the flow of a non-Newtonian fluid past an infinite plate, Meccanica. 19 (1984) 158–160.
[27] J.B. Mcleod, K.R. Rajagopal, On the uniqueness of flow of a Navier–Stokes fluid due to a stretching boundary, Arch. Ration. Mech. Anal. 98 (1987) 385–395.
[28] P.S. Lawrence, B.N. Rao, The non-uniqueness of the MHD flow of a viscoelastic fluid past a stretching sheet, Acta Mech 112 (1995) 223–228.
[29] M. Abramowitz, L.A. Stegun, Handbook of mathematical functions, National Bureau of Standards/Amer. Math. Soc. 55 (1972), providence, RI.
[30] R.M. Sonth, S.K. Khan, M.S. Abel, K.V. Prasad, Heat and mass transfer in a viscoelastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation, Heat and Mass transfer 38 (2002) 213–230.